Original Article

Journal of Iranian Medical Council, Volume 9, Issue no. 4
http://dx.doi.org/10.18502/jimc.v9i4.22459

A Mathematical Model to Predict the Angle of Z-plasty and Rotation with Trigonometric Relationships

Fardin Rahmanpoor[1,2], Tooraj Zandbaf[3] and Ali Ahmadabadi[4*]

1. Innovative Medical Research Center, Islamic Azad University, Mashhad, Iran
2. Department of Mathematics, Yazd University, Yazd, Iran
3. Department of General Surgery, Islamic Azad University, Mashhad, Iran
4. Surgical Oncology Research Centre, Mashhad University of Medical Sciences, Mashad, Iran

*** Corresponding author**

Ali Ahmadabadi, MD

Surgical Oncology Research Centre, Mashhad University of Medical Sciences, Mashad, Iran

Tel: +98 9151257319

Email: ahmadabadia@mums.ac.ir

Received: 19 May 2025

Accepted: 27 Oct 2025

Citation to this article

Rahmanpoor F, Zandbaf T, Ahmadabadi A. A Mathematical Model to Predict the Angle of Z-plasty and Rotation with Trigonometric Relationships. *J Iran Med Counc*. 2026;9(4):908-13.

Abstract

Background: Burns contractions restrict limb movement. Z-plasty is a widely used technique for repairing contractures resulting from burns.

Methods: Equations were derived utilizing trigonometric relationships that enable surgeons to estimate the angle and rotation of the Z-plasty based on the degree of lengthening required for the contracted area.

Results: This study introduces a mathematical model designed to assist surgeons in selecting a Z-plasty with the appropriate angle and rotation. Numerical results were presented for various lengths of contracture in non-classical Z-plasties.

Conclusion: This model aims to enhance surgical precision and facilitate the development of standardized procedures.

Keywords: Burn, Contracture, Mathematical Model, Z-Plasty

Introduction

Wound repair involves fibroblast proliferation, extracellular matrix production, and tissue remodeling (1). Depending on the location of a skin wound, the reconstructed tissue may experience undesirable deformities (2). In severe cases, hypertrophic tissues form, necessitating medical and surgical rehabilitation intervention to restore function and aesthetics (3). Scar contracture following a burn injury poses significant challenges that impair the patient's functional abilities. The primary cause of contractures after burns is often inadequate medical attention, rehabilitation, and insufficient treatment. Z-plasty is a precise technique in plastic surgery that leverages the elasticity of the tissue surrounding a scar (4), determined through qualitative and quantitative assessments during a clinical examination. Z-plasty can be utilized in the following scenarios: i) aligning and altering the structure of tissues affected by stair-step deformity, where transferring tissue to a new position is advantageous, and ii) releasing contractions by lengthening the scar along its axis, thereby reducing tension on the scar. Classic Z-plasty surgery can be performed in two ways: by changing the size of the limb or adjusting the angle (3). Increasing the angle and enlarging the flaps can result in a tension increase of 7 to 10 times required for closure (5), making flap transfer challenging and leading to complications such as dog ears(3). Conversely, a slight angle may shorten the base of the flap, weakening vascular flow and leading to flap ischemia and necrosis. In Z-plasty, the triangular transposition flap is commonly employed, typically adjusted symmetrically to a 60-degree angle, necessitating a 75% increase in the length of the scar limb (2). Transposition flaps are frequently utilized in plastic surgery, especially in burn cases, and surgeons may have differing opinions on the ideal angle and rotation of the flap (3).

This study aimed to introduce a novel formula for determining the appropriate angle and rotation using mathematical relationships, potentially establishing a standardized procedure for repairing contractures and diminishing reliance on the surgeon's experience. In conventional approaches, the design of a classical Z-plasty was employed, in which predetermined angles and limb lengths were utilized, and trigonometric relationships were applied to demonstrate the achievable gain. In contrast, the method was proposed is tailored to the patient's clinical context and the surgeon's specific objective regarding the extent of contracture release required to optimize the patient's range of motion. By selecting the desired or appropriate gain and applying trigonometric calculations, a novel Z-plasty configuration with newly defined angles and limb lengths is established. Although the mathematical foundation resembles that of previous methods, the practical application is distinct, as it provides a customized design while maintaining the fundamental standards of a classical Z-plasty. This provides a more accurate, reproducible, and clinically applicable framework compared to earlier theoretical models.

Matereials and Method

As with all restorative surgical techniques, measurement accuracy and an appropriate model are essential in Z-plasty (6). The fundamental basis of Z-plasty consists of two triangular flaps associated with transposition and rotation (7). In classical Z-plasty, incisions of equal length create two symmetric triangular flaps (8). A central limb can either be positioned over the scar or parallel to it, with two arms originating at different angles from this central limb; all three limbs are usually equal in length (9). After dissection, the two flaps are rotated in opposite directions to cover the remaining defect, if XY represents the scar.

The point where the two common limbs intersect is referred to as point O. The angle XOA, shown as β in the diagram, measures the degree of rotation of the common limb in a Z-plasty.

In figure 1, XY represents the scar. While lines AX and BY are drawn so that angles AXY and XYB equal θ°, resulting in flaps 1 and 2. Figure 2 illustrates the position of flaps 1 and 2 after Z-plasty. To apply trigonometric principles to our model, the dashed line XY, indicating the original scar line, is depicted in figure 3. Thus, the scar line XY represents the common limb of the Z-plasty before flap transposition, while line AB represents the position of the common limb after flaps 1 and 2 are transposed. The point where the two common limbs intersect is referred to as point O. The angle XOA, donated as β in the diagram,

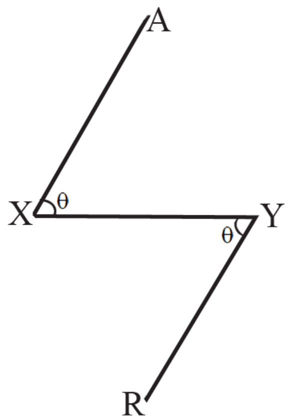

Figure 1. The scar line XY represents the common limb before flap transposition.

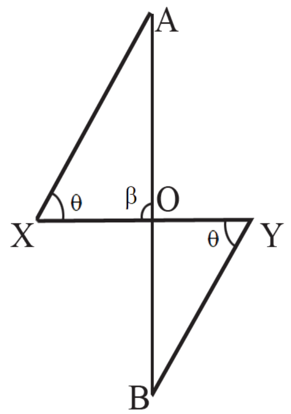

Figure 2. β present the angle of rotation.

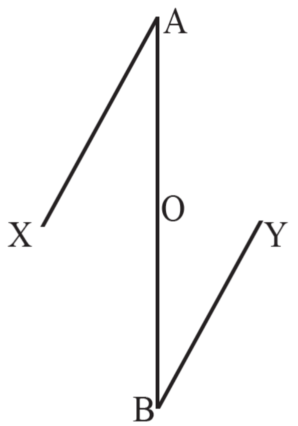

Figure 3. The line AB show the position of the common limb after transposition flaps.

measures the degree of rotation of the common limb in Z-plasty. Next, consider the following:

$$\mathrm{AB} = \mathrm{L}(cm)$$

$$\mathrm{AX} = \mathrm{BY} = \mathrm{XY} = \mathrm{T}(cm)$$

Using trigonometric relationships (6,10), the desired angle and rotation was determined by calculating the appropriate increase in the length of contraction:

$$AY^2 = XY^2 + AX^2 - 2(XY)(AX)\cos\theta$$

$$(\frac{L}{2})^2 = T^2 + T^2 - 2T^2\cos\theta$$

$$\frac{L^2}{4} = 2T^2 - 2T^2\cos\theta$$

The angle in a Z-plasty can be calculated as follows:

$$\theta = Arc\cos(1 - \frac{L^2}{8T^2})$$

The angle of rotation can be determined using the following relationship.

$$AX^2 = XO^2 + AO^2 - 2(XO)(AXO)\cos\beta$$

$$T^2 = \left(\frac{L}{2}\right)^2 + \left(\frac{T}{2}\right)^2 - \left(\frac{TL}{2}\right)\cos\beta$$

$$\frac{3}{4}T^2 = \frac{L^2}{4} - \left(\frac{2TL}{4}\right)\cos\beta$$

$$\cos\beta = \frac{L^2 - 3T^2}{2TL}$$

$$\beta = Arc\cos(\frac{L^2 - 3T^2}{2TL})$$

Results

As shown in the equations, the diagonal AB and the common limb (T) lengths are given. The size of the angles θ and β can be derived from equations 1 and 2. The biomechanics of skin tension may affect the mathematical predictions for estimating angles θ and β after completing the Z-plasty (5). It is known that the limbs of the Z can extend at various angles ranging from 30° to 90° (8). This study is mathematical; however, the skin doesn't behave as a mathematical model, as Furnas *et al* (5) confirmed. It is essential to find a method to accurately measure the change in the direction of the common limb in the skin (11). Such a method would assist surgeons in understanding the relationship between the size of the defect, the angle, and the rotation of Z-plasty, both before and after the procedure, according to the length of the scar. A mathematical analysis of the degree of rotation

Table 1. The degree of angle Z and rotation for various length of contracture

Percent Increase length of contracture	Angle	Angle of rotation
12.5%	36	125
25%	40	114
37.5%	44	104
50%	48	96
62.5%	52	94
75%	56	92
79.5%	60	90
100%	64	69
112.5%	68	63
125%	73	56
137.5%	77	49
150%	82	42
162.5%	87	34

for various angles of Z-plasty was conducted, with results tabulated in table 1.

With careful planning and mastery in executing Z-plasty, impressive results can be achieved in alleviating contractures and limitations in the movement of limbs by reconstructing tissue and utilizing appropriately sized flaps (12).

Discussion

In 1837, Horners reported at the University of Pennsylvania on the ectropion of the left lower eyelid due to a burn scar (13). Denoiville performed the first Z-plasty on the right lower eyelid in 1863 (14). In 1904, Berger described Z-plasty as a technique involving the displacement of flaps to treat burn complications (15). In 1914, Morstein corrected a complication from finger flexion using Z-plasty (16). Limberg enhanced the flap design by introducing mathematical concepts (17), which were further developed by Davis (18). In 1963, McGregor performed Z-plasty on hand skin contractures and significantly popularized the technique (19). The method introduced in this study defines the optimal Z-angle and appropriate rotation, considering the desired increase in length as determined by the surgeon. Unlike previous methods, which relied on predetermined angles and limb lengths to demonstrate the expected gain in a classical Z-plasty, the author's proposed approach is based on the patient's clinical condition and the surgeon's objective. By defining new angles and limb lengths through trigonometric calculations, the technique maintains the standards of the classical Z-plasty while offering a more tailored and clinically relevant design.

However, the geometrical principles were first elegantly described by Alexander Limberg in 1929 (17). The results of the present study align with these principles of Alexander Limberg, suggesting that the present study findings may provide guidance for plastic surgery using the Z-plasty technique (Chart 1). Schematic redrawing by the authors based on the original geometric description of Limberg (17). The plotted data correspond to the calculations performed in this study and illustrate the consistency between

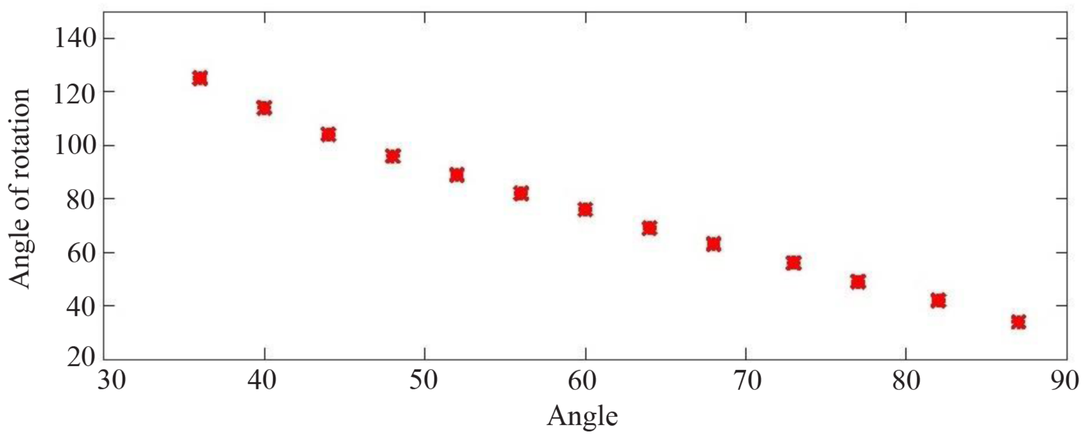

Chart 1. Schematic redrawing by the authors based on the original geometric description of Limberg

computational results and graphical representation. This study is based primarily on mathematical modeling and geometrical analysis, which do not account for individual variations in tissue perfusion, elasticity, or intraoperative conditions. The surgeon's clinical judgment plays a critical role in flap selection, design and intraoperative assessment of tissue perfusion and flexibility. These factors are indeed essential determinants of surgical outcomes. However, incorporating such qualitative and intraoperative variables into mathematical models requires further investigations and methodological development. This limitation has been acknowledged in the present study.

Conclusion

This paper proposes a new model based on trigonometric relationships to determine the angle size and Z-rotation. The primary advantage of the proposed model is that it simplifies the management of increased contraction length, ensuring that equations 1 and 2 always provide a unique solution. The concept behind this new model holds potential for future use. We implemented this model using MATLAB.

Acknowledgement

This work was supported by the Islamic Azad University, Mashhad, Iran. The study was approved by the Ethics Committee of the Islamic Azad University, Mashhad, Iran, with ethical approval code: IR.IAU. MSHD.REC.1403.061.

Conflict of Interest

Authors declare no conflict of interest.

References

1. Brunicardi F, Andersen D, Billiar T, Dunn D, Hunter J, Matthews J, et al. Schwartz's principles of surgery, 10e: McGraw-hill; 2014.

2. Matsumoto E, Liang H, Mahadevan L. Topology, geometry and mechanics of surgical Z-plasty. arXiv preprint arXiv:190500801. 2019.

3. Golder B, Kabir H, Sohelullah M. Z-Plasty with Skin Grafting for Post-Burn Contracture-Study of 30 Cases. Sch J App Med Sci 2022;8:1104-98.

4. Hayashida K, Akita S. Surgical treatment algorithms for post-burn contractures. Burns Trauma 2017 Mar 14:5:9.

5. Greenstein G, Greenstein B, Cavallaro J, Elian N, Tarnow D. Flap advancement: practical techniques to attain tension-free primary closure. J Periodontol 2009 Jan;80(1):4-15.

6. Lima FC, Lessio BP, LIMA GAD, AMARAL TPA, Abramo AC. Z-plasty: a comparative study between theoretical and practical elongation. Revista Brasileira de Cirurgia Plástica 2023;31:540-4.

7. Thomas JR, Somenek M. Scar revision review. Arch Facial Plast Surg 2012 May-Jun;14(3):162-74.

8. Sunderraj E. A mathematical model to predict the requirement for multiple Z plasties. Indian J Plast Surg 2008 Jan;41(1):99-101.

9. Aasi SZ. Z-plasty made simple. Dermatol Res Pract 2010:2010:982623.

10. Andreasen S. Tasks for Learning Trigonometry. 2024.

11. Huang C, Ogawa R. Three-dimensional reconstruction of scar contracture-bearing axilla and digital webs using the square flap method. Plast Reconstr Surg Glob Open 2014 Jun 6;2(5):e149.

12. Anastasova V, Kiskinov P, Georgiev A, Ivanova K, Krasteva E, Atliev K, et al. Z-plasty–Basic Surgical Technique for Post-burn Patients. Acta Medica Bulgarica 2025;52(1):21-8.

13. Horner WE, ART V. Clinical Report on the Surgical Department of the Philadelphia Hospital, Blockley, for the months of May, June, and July, 1837. The American Journal of the Medical Sciences (1827-1924) 1837;21(41):99.

14. Denonvilliers C. De la méthode autoplastique par pivotempar appliquée à la restauration des paupières. Bull Gén Thér Méd Chirurg 1863;65:110-22.

15. Berger P. Autoplastie par dédoublement de la palmure et échange de lambeaux. Chirurgie ortopédique Paris: Steinfeil 1904:180-5.

16. Morestin H. De la correction des flexions permanentes des doigts. Rev Chir 1914;50:1-27.

17. Limberg A. Skin plastic with shifting triangle flaps. Collect Sci Works Mem Of 1929;50:862.

18. Davies D. The repair of the unilateral cleft lip. Br J Plast Surg 1965 Jul;18:254-64.

19. McGregor IA, MCCORMACK RM. Fundamental techniques of plastic surgery and their surgical applications. Plastic and Reconstructive Surgery 1963;31(2):192.